



the function $f(x, y) = \sin(xy)$ plotted by computer

2 Functions

Throughout most of this book we're going to be using a technique called functional programming. We can't give a complete definition of this term yet, but in this chapter we introduce the guiding idea of function programming: the function.

Basically, every function does one thing that your high school geometry teacher might accept: that our functions don't necessarily relate to numbers. But the essential idea is just like the kind of function described by $f(x) = x - 1$: in that the input is the name of a function that function takes an argument called x , which is a number and returns some other number.

In this chapter you're going to use the computer to explore functions, but you're not going to use the standard Scheme notation as in the rest of the book. What's special about this chapter is that we want to separate the idea of functions from the complexities of programming language notation. For example, Scheme notation lets you write expressions that in some cases use more than one function, but in this chapter you can only use one at a time.

To get into this chapter's special computer interface, first start running Scheme as you did in the first chapter, then type

```
(load "functions.scm")
```

to tell Scheme to read the program you're using. If you have trouble loading the program, look in Appendix A for further information. After loading, then to start the program, type

```
(functions)
```

You then see the computer carry out interactions like the following in the text editor. The computer prints what you type in **boldface** and what the computer types in **lightface** printing

```
Function: +  
Argument: 3  
Argument: 5
```

```
The result is: 8
```

```
Function: sqrt  
Argument: 144
```

```
The result is: 12
```

As you can see, different functions can have different numbers of arguments. In these examples, we added two numbers and we took the square root of one number. However, every function gives exactly one result each time we use it.

Now we'll see the functions program type `exit` when it's done for a function.

Arithmetic

Experiment with these arithmetic functions: `+`, `-`, `*`, `/`, `sqrt`, `quotient`, `remainder`, `random`, `round`, `max`, and `expt`. Try different kinds of numbers, including integers and numbers with decimal fractions. Watch what happens if you try to divide by zero. Throughout this chapter, we're going to let you experiment with functions rather than just give you a long boring list of how each one works. The boring list is in the appendix for reference on page 18.

Try these

```
Function: /  
Argument: 1  
Argument: 987654321987654321
```

```
Function: remainder  
Argument: 12  
Argument: -5
```

If you get no response after you type `(functions)`, just press the Return or Enter key. In the next section, your instructor will read Appendix A to see how to do this.

Function: round
Argument: 17.5

These are just a few suggestions. Be creative, don't just type in our examples.

Words

Not Scheme functions deal with numbers. A broader category of argument is the word, including numbers but also including English words like spaghetti or xylophone. Even the meaningless sequence of letters and digits such as glo87rp is considered a word. Try these functions that accept words as arguments: `first`, `butfirst`, `last`, `butlast`, `word`, and `count`. What happens if you use a number as the argument to one of these?

Function: butfirst
Argument: a

Function: count
Argument: 765432

So far most of our functions fall into one of two categories: the arithmetic functions which require numbers as arguments and return numbers as the result, and the word functions which accept words as arguments and return words as the result. The one exception we've seen is `count`, which kind of argument does `count` accept? What kind of value does it return? The technique for finding out is type.

In principle you could think of almost anything's type such as numbers that contain the digit 7. Such ad hoc types are legitimate and sometimes useful, but there are so few of them that Scheme doesn't support them. Types can't be represented by numbers, so they're not considered words.

Function: word
Argument: 3.14
Argument: 1592654

Function: +
Argument: 6
Argument: seven

Certain punctuation characters can't be used in words, but let's defer the details until you've gotten to know the word functions with simplicity.

Domain and Range

The technique for the things that a function accepts as an argument is the domain of the function. The name for the things that a function returns is its range. So the domain of `count` is words and the range of `count` is numbers. In fact, nonnegative integers. This example shows that the range is not exactly one of our standard data types: there is no nonnegative integer type in Scheme.

How do you tell out the domain and range of a function? You could say for example, the `cos` function has numbers as its domain and numbers between $-\pi$ and π as its range. Or informally you may say `cos` takes numbers as its argument and returns a number between $-\pi$ and π .

For functions of two or more arguments the language is a little less straightforward. The informal version still works. `Remainder` takes two integers as arguments and returns an integer. But you can't say the domain of `remainder` is two integers because the domain of a function is the set of possible arguments, not just the set that output the characteristics of arguments.

By the way, remember that in simplifications in this chapter. For example, Scheme's `+` function can actually accept any number of arguments, not just two. But we don't want to go into these details until we have a history lesson, so we start with adding two numbers to this.

Here are examples that illustrate the domains of some functions.

```
Function: expt
Argument: -3
Argument: .5
```

```
Function: expt
Argument: -3
Argument: -3
```

```
Function: remainder
Argument: 5
Argument: 0
```

Remember your version of Scheme has some special numbers.

Remember the technique says the domain of `remainder` is the Cartesian cross product of the integers and the integers in order to avoid that foolishness. Just use the informal reasoning.

More Types: Sentences and Booleans

We're going to introduce more data types and more functions that include those types in their domain or range. The next type is the sentence: a bunch of words enclosed in parentheses such as

`(all you need is love)`

Don't include any punctuation characters within the sentence. Many of the functions that accept words in their domain also accept sentences. Here is so function sentence that accepts words and sentences. `rye` `pes` `ie` `butfirst` of sentence

Function: `sentence`

Argument: `(when i get)`

Argument: `home`

Function: `butfirst`

Argument: `(yer blues)`

Function: `butlast`

Argument: `()`

Other important functions are used to ask yes or no questions. That is the range of these functions contains only two values: one meaning true and the other meaning false. Try the numeric comparisons `=` `<` `>` `<=` and `>=` and the functions `equal?` and `member?` that work on words and sentences. The question mark is part of the name of the function. Here are so functions `and` `or` and `not` whose domain and range are both true/false values. The two values true and false are called Booleans, named after George Boole. He developed the formal tools used for true/false values in the circuits.

It's good to have these true/false values. Often a program must choose between two options. If the number is positive do this; if negative do that. Scheme has functions to make such choices based on true/false values. For now you can experiment with the `if` function. Its first argument must be true or false; the others can be anything.

Our Favorite Type: Functions

So far our data types include numbers words sentences and Booleans. Scheme has several more data types, but for now let's just consider one more. A function can be used as data. Here's one example.

Function: number-of-arguments
Argument: equal?

The result is: 2

The range of `number-of-arguments` is nonnegative integers. But its domain is functions. For example, try using it as an argument to itself:

If you've used other computer programming languages, it may seem strange to use a function that is part of a computer program as data. Most languages emphasize the distinction between program and data. We soon see that the ability to treat functions as data helps the programmer writing every powerful and convenient

try these examples

Function: `every`
Argument: `first`
Argument: (the long and winding road)

Function: `keep`
Argument: `vowel?`
Argument: `constantinople`

When checking out these, you're not applying the function `first` to the sentence (the long and winding road); you're applying the function `every` to a function and sentence.

Other functions that can be used with `keep` include `even?` and `odd?`. These do things to the integers and `number?` does do anything.

Play with It

If you've been reading the book without trying things out on the computer, you go along getting to work. Spend some time getting used to these ideas and thinking about them when you're done reading the book.

Thinking about What You've Done

The idea of function is at the heart of both the mathematics and computer science. For example, when the mathematicians first thought about the system of numbers, they used functions to create the integers. They say that's suppose we have one number

called zero then let's suppose we have the function given by $f(x) = x$. By applying that function repeatedly we can create f_1 then f_2 and so on.

Functions are important in computer science because they give us a way to think about process in simple English. A way to think about something happening so something changing. A function embodies transformation of information taking in something and returning something else didn't notice that computers do that they transform information to produce new results.

A lot of the techniques taught in school is about numbers but we've seen that functions don't have to be about numbers. We've used functions of words and sentences such as **first** and even functions of functions such as **keep**. You can imagine functions that transform information of any kind, such as the function French → Indo → fenetre or the function capital → C → ifornia → Sacramento.

You've done a lot of thinking about the domain and range of functions. You can add to numbers but it doesn't make sense to add to words that represent numbers. So for argument functions have to be composed of domains. We use the concept of values for one argument depend on the specific value used for the other one. The function **exp** is a nice example. Make sure you've tried both positive and negative numbers and fractions. See how the number powers.

Part of the definition of a function is that you always get the same answer whenever you call a function with the same argument. The value returned by the function in other words shouldn't change regardless of anything else you say. The computer doesn't know. One of the functions you've explored in this chapter isn't really a function according to this rule. Which one? The rule you see is too restrictive and indeed it's often convenient to use the name function loosely for processes that can give different results in different circumstances. But we see that sometimes it's important to stick with the strict definition and refrain from using processes that aren't truly functions.

We've hinted at two different ways of thinking about functions. The first is called function as process. Here a function is a rule that tells us how to transform some information into some other information. The function is just a rule not a thing in its own right. The other things are the words or numbers or whatever the function manipulates. The second way of thinking is called function as object. In this case a function is a perfectly good thing in itself. We can use functions as arguments to another function for example. Research with college students shows that this second idea is hard for most people but it's worth the effort because you will see that higher-order functions, functions of functions like **keep** and **every** can make programs such as easier to write.

As how they naturally think about a root peeler, we focus our attention on the carrots which refer to the intent to eat, then the peeler just represents process. We're peeling carrots, we're applying the function `peel` to carrots, it's the carrot that counts. But we can also think about the peeler's thing in its own right when we can't worry about whether its blade is sharp enough.

We might think that we haven't explored in this chapter, though we used it a lot in Chapter 1. This is the composition of functions using the result from one function as an argument to another function. It's crucial to write large programs by doing a bunch of small functions and then composing them with each other to produce the desired result. We'll start doing that in the next chapter. Here we return to the Scheenotion.

Exercises

Use the `functions` program for all these exercises.

2.1 In each line of the following table, delete one piece of information. Fill in the missing details.

function	arg	arg	result
<code>word</code>	<code>now</code>	<code>here</code>	
<code>sentence</code>	<code>now</code>	<code>here</code>	
<code>first</code>	<code>blackbird</code>	<code>none</code>	
<code>first</code>	<code>(blackbird)</code>	<code>none</code>	
	<code>3</code>	<code>4</code>	<code>7</code>
<code>every</code>		<code>(thank you girl)</code>	<code>(thank you girl)</code>
<code>member?</code>	<code>e</code>	<code>aardvark</code>	
<code>member?</code>	<code>the</code>		<code>#t</code>
<code>keep</code>	<code>vowel?</code>	<code>(i will)</code>	
<code>keep</code>	<code>vowel?</code>		<code>eieio</code>
<code>last</code>	<code>()</code>	<code>none</code>	
	<code>last</code>	<code>(honey pie)</code>	<code>(y e)</code>
		<code>taxman</code>	<code>aa</code>

2.2 What is the domain of the `vowel?` function?

Yes, there is an English word that's too difficult to pronounce.

2.3 One of the functions you can use is called `appearances`. Experiment with it and then describe fully its domain and range and what it does. Measure to try lots of cases. Hint: think about its name.

2.4 One of the functions you can use is called `item`. Experiment with it and then describe fully its domain and range and what it does.

The following exercises are for functions that meet certain criteria. For your convenience here are the functions in this chapter: `+`, `-`, `/`, `<=`, `<`, `=`, `>=`, `>` and `appearances` but `first`, `butlast`, `cos`, `count`, `equal?`, `every`, `even?`, `expt`, `first`, `if`, `item`, `keep`, `last`, `max`, `member?`, `not`, `number?`, `number-of-arguments`, `odd?`, `or`, `quotient`, `random`, `remainder`, `round`, `sentence`, `sqrt`, `vowel?`, and `word`.

2.5 List the one argument functions in this chapter for which the type of the return value is always different from the type of the argument.

2.6 List the one argument functions in this chapter for which the type of the return value is sometimes different from the type of the argument.

2.7 Make the students sometimes use the term "operator" to mean "function of two arguments" both of the same type that returns a result of the same type. Which of the functions you've seen in this chapter satisfy that definition?

2.8 An operator `f` is commutative if `f a b` is `f b a` for possible arguments `a` and `b`. For example `+` is commutative but `word` isn't. Which of the operators from Exercise 1 are commutative?

2.9 An operator `f` is associative if `f f a b c` is `f a f b c` for possible arguments `a`, `b`, and `c`. For example `*` is associative but `/` is not. Which of the operators from Exercise 1 are associative?