Trombone players produce different pitches partly by varying the length of a tube.

7 Variables

A *variable* is a connection between a name and a value.* That sounds simple enough, but some complexities arise in practice. To avoid confusion later, we'll spend some time now looking at the idea of "variable" in more detail.

The name *variable* comes from algebra. Many people are introduced to variables in high school algebra classes, where the emphasis is on solving equations. "If $x^3 - 8 = 0$, what is the value of x?" In problems like these, although we call x a variable, it's really a *named constant!* In this particular problem, x has the value 2. In any such problem, at first we don't know the value of x, but we understand that it does have some particular value, and that value isn't going to change in the middle of the problem.

In functional programming, what we mean by "variable" is like a named constant in mathematics. Since a variable is the connection between a name and a value, a formal parameter in a procedure definition isn't a variable; it's just a name. But when we invoke the procedure with a particular argument, that name is associated with a value, and a variable is created. If we invoke the procedure again, a *new* variable is created, perhaps with a different value.

There are two possible sources of confusion about this. One is that you may have programmed before in a programming language like BASIC or Pascal, in which a variable often *does* get a new value, even after it's already had a previous value assigned to it. Programs in those languages tend to be full of things like "`X = X + 1`." Back in Chapter 2 we told you that this book is about something called "functional programming," but we haven't yet explained exactly what that means. (Of course we *have* introduced a lot

* The term "variable" is used by computer scientists to mean several subtly different things. For example, some people use "variable" to mean just a holder for a value, without a name. But what we said is what *we* mean by "variable."

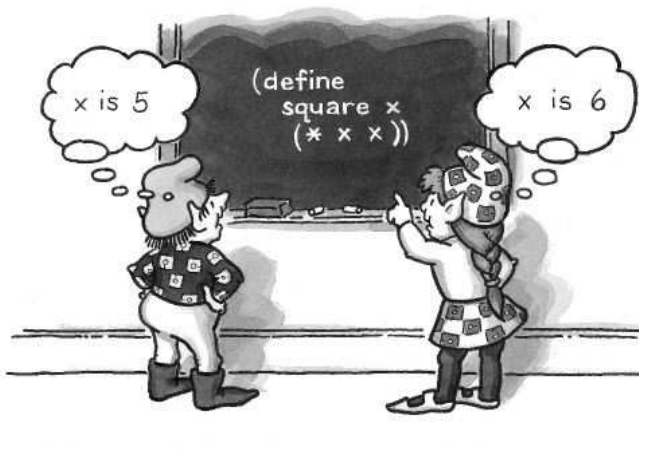
of functions, and that is an important part of it.) Part of what we mean by functional programming is that once a variable exists, we aren't going to *change* the value of that variable.

The other possible source of confusion is that in Scheme, unlike the situation in algebra, we may have more than one variable with the same name at the same time. That's because we may invoke one procedure, and the body of that procedure may invoke another procedure, and each of them might use the same formal parameter name. There might be one variable named `x` with the value 7, and another variable named `x` with the value 51, at the same time. The pitfall to avoid is thinking "`x` has changed its value from 7 to 51."

As an analogy, imagine that you are at a party along with Mick Jagger, Mick Wilson, Mick Avory, and Mick Dolenz. If you're having a conversation with one of them, the name "Mick" means a particular person to you. If you notice someone else talking with a different Mick, you wouldn't think "Mick has become a different person." Instead, you'd think "there are several people here all with the name Mick."

How Little People Do Variables

You can understand variables in terms of the little-people model. A variable, in this model, is the association in the little person's mind between a formal parameter (name) and the actual argument (value) she was given. When we want to know `(square 5)`, we hire Srini and tell him his argument is 5. Srini therefore substitutes 5 for `x` in the body of `square`. Later, when we want to know the square of 6, we hire Samantha and tell her that her argument is 6. Srini and Samantha have two different variables, both named `x`.



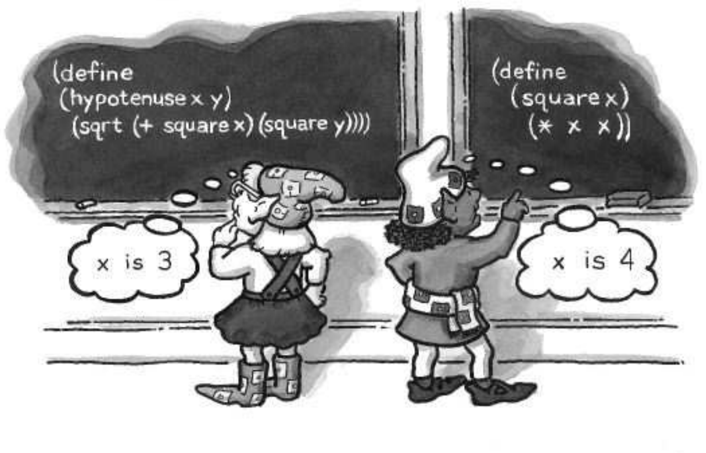
Srini and Samantha do their work separately, one after the other. But in a more complicated example, there could even be more than one value called x at the same time:

```
(define (square x) (* x x))

(define (hypotenuse x y)
  (sqrt (+ (square x) (square y))))

> (hypotenuse 3 4)
5
```

Consider the situation when we've hired Hortense to evaluate that expression. Hortense associates the name x with the value 3 (and also the name y with the value 4, but we're going to pay attention to x). She has to compute two squares. She hires Solomon to compute (square 3). Solomon associates the name x with the value 3. This happens to be the same as Hortense's value, but it's still a separate variable that could have had a different value—as we see when Hortense hires Sheba to compute (square 4). Now, simultaneously, Hortense thinks x is 3 and Sheba thinks x is 4.



(Remember that we said a variable is a connection between a name and a value. So x isn't a variable! The association of the name x with the value 5 is a variable. The reason we're being so fussy about this terminology is that it helps clarify the case in which several variables have the same name. But in practice people are generally sloppy about this fine point; we can usually get away with saying "x is a variable" when we mean "there is some variable whose name is x.")

Another important point about the way little people do variables is that they can't read each others' minds. In particular, they don't know about the values of the local variables that belong to the little people who hired them. For example, the following attempt to compute the value 10 won't work:

```
(define (f x)
  (g 6))

(define (g y)
  (+ x y))

> (f 4)
ERROR -- VARIABLE X IS UNBOUND.
```

We hire Franz to compute `(f 4)`. He associates `x` with 4 and evaluates `(g 6)` by hiring Gloria. Gloria associates `y` with 6, but she doesn't have any value for `x`, so she's in trouble. The solution is for Franz to tell Gloria that `x` is `4`:

```
(define (f x)
  (g x 6))

(define (g x y)
  (+ x y))

> (f 4)
10
```

Global and Local Variables

Until now, we've been using two very different kinds of naming. We have names for procedures, which are created permanently by `define` and are usable throughout our programs; and we have names for procedure arguments, which are associated with values temporarily when we call a procedure and are usable only inside that procedure.

These two kinds of naming seem to be different in every way. One is for procedures, one for data; the one for procedures makes a permanent, global name, while the one for data makes a temporary, local name. That picture does reflect the way that procedures and other data are *usually* used, but we'll see that really there is only one kind of naming. The boundaries can be crossed: Procedures can be arguments to other procedures, and any kind of data can have a permanent, global name. Right now we'll look at that last point, about global variables.

Just as we've been using `define` to associate names with procedures globally, we can also use it for other kinds of data:

```
> (define pi 3.141592654)

> (+ pi 5)
8.141592654

> (define song '(I am the walrus))

> (last song)
WALRUS
```

Once defined, a global variable can be used anywhere, just as a defined procedure can be used anywhere. (In fact, defining a procedure creates a variable whose value is the procedure. Just as `pi` is the name of a variable whose value is 3.141592654, `last` is the name of a variable whose value is a primitive procedure. We'll come back to this point in Chapter 9.) When the name of a global variable appears in an expression, the corresponding value must be substituted, just as actual argument values are substituted for formal parameters.

When a little person is hired to carry out a compound procedure, his or her first step is to substitute actual argument values for formal parameters in the body. The same little person substitutes values for global variable names also. (What if there is a global variable whose name happens to be used as a formal parameter in this procedure? Scheme's rule is that the formal parameter takes precedence, but even though Scheme knows what to do, conflicts like this make your program harder to read.)

How does this little person know what values to substitute for global variable names? What makes a variable "global" in the little-people model is that *every* little person knows its value. You can imagine that there's a big chalkboard, with all the global definitions written on it, that all the little people can see. If you prefer, you could imagine that whenever a global variable is defined, the `define` specialist climbs up a huge ladder, picks up a megaphone, and yells something like "Now hear this! `Pi` is 3.141592654!"

The association of a formal parameter (a name) with an actual argument (a value) is called a *local variable.*

It's awkward to have to say "Harry associates the value 7 with the name `foo`" all the time. Most of the time we just say "`foo` has the value 7," paying no attention to whether this association is in some particular little person's head or if everybody knows it.

The Truth about Substitution

We said earlier in a footnote that Scheme doesn't actually do all the copying and substituting we've been talking about. What actually happens is more like our model of global variables, in which there is a chalkboard somewhere that associates names with values—except that instead of making a new copy of every expression with values substituted for names, Scheme works with the original expression and looks up the value for each name at the moment when that value is needed. To make local variables work, there are several chalkboards: a global one and one for each little person.

The fully detailed model of variables using several chalkboards is what many people find hardest about learning Scheme. That's why we've chosen to use the simpler substitution model.*

`Let`

We're going to write a procedure that solves quadratic equations. (We know this is the prototypical boring programming problem, but it illustrates clearly the point we're about to make.)

We'll use the quadratic formula that you learned in high school algebra class:

$$ax^2 + bx + c = 0 \quad \text{when} \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

```
(define (roots a b c)
  (se (/ (+ (- b) (sqrt (- (* b b) (* 4 a c))))
         (* 2 a))
      (/ (- (- b) (sqrt (- (* b b) (* 4 a c))))
         (* 2 a))))
```

Since there are two possible solutions, we return a sentence containing two numbers. This procedure works fine,** but it does have the disadvantage of repeating a lot of the

* The reason that all of our examples work with the substitution model is that this book uses only functional programming, in the sense that we never change the value of a variable. If we started doing the `X = X + 1` style of programming, we would need the more complicated chalkboard model.

** That is, it works if the equation has real roots, or if your version of Scheme has complex work. It computes the square root part of the formula twice. We'd like to avoid that inefficiency.

One thing we can do is to compute the square root and use that as the actual argument to a helper procedure that does the rest of the job:

```
(define (roots a b c)
  (roots1 a b c (sqrt (- (* b b) (* 4 a c)))))

(define (roots1 a b c discriminant)
  (se (/ (+ (- b) discriminant) (* 2 a))
      (/ (- (- b) discriminant) (* 2 a))))
```

This version evaluates the square root only once. The resulting value is used as the argument named `discriminant` in `roots1`.

We've solved the problem we posed for ourselves initially: avoiding the redundant computation of the discriminant (the square-root part of the formula). The cost, though, is that we had to define an auxiliary procedure `roots1` that doesn't make much sense on its own. (That is, you'd never invoke `roots1` for its own sake; only `roots` uses it.)

Scheme provides a notation to express a computation of this kind more conveniently. It's called `let`:

```
(define (roots a b c)
  (let ((discriminant (sqrt (- (* b b) (* 4 a c)))))
    (se (/ (+ (- b) discriminant) (* 2 a))
        (/ (- (- b) discriminant) (* 2 a)))))
```

Our new program is just an abbreviation for the previous version: In effect, it creates a temporary procedure just like `roots1`, but without a name, and invokes it with the specified argument value. But the `let` notation rearranges things so that we can say, in the right order, "let the variable `discriminant` have the value `(sqrt...)` and, using that variable, compute the body."

`Let` is a special form that takes two arguments. The first is a sequence of name-value pairs enclosed in parentheses. (In this example, there is only one name-value pair.) The second argument, the *body* of the `let`, is the expression to evaluate.

---

numbers. Also, the limited precision with which computers can represent irrational numbers can make this particular algorithm give wrong answers in practice even though it's correct in theory.

Now that we have this notation, we can use it with more than one name-value connection to eliminate even more redundant computation:

```
(define (roots a b c)
  (let ((discriminant (sqrt (- (* b b) (* 4 a c))))
        (minus-b (- b))
        (two-a (* 2 a)))
    (se (/ (+ minus-b discriminant) two-a)
        (/ (- minus-b discriminant) two-a))))
```

In this example, the first argument to `let` includes three name-value pairs. It's as if we'd defined and invoked a procedure like the following:

```
(define (roots1 discriminant minus-b two-a) ...)
```

Like `cond`, `let` uses parentheses both with the usual meaning (invoking a procedure) and to group sub-arguments that belong together. This grouping happens in two ways. Parentheses are used to group a name and the expression that provides its value. Also, an additional pair of parentheses surrounds the entire collection of name-value pairs.

Pitfalls

⇒ If you've programmed before in other languages, you may be accustomed to a style of programming in which you *change* the value of a variable by assigning it a new value. You may be tempted to write

```
> (define x (+ x 3))                          ;; no-no
```

Although some versions of Scheme do allow such redefinitions, so that you can correct errors in your procedures, they're not strictly legal. A definition is meant to be *permanent* in functional programming. (Scheme does include other mechanisms for non-functional programming, but we're not studying them in this book because once you allow reassignment you need a more complex model of the evaluation process.)

⇒ When you create more than one temporary variable at once using `let`, all of the expressions that provide the values are computed before any of the variables are created. Therefore, you can't have one expression depend on another:

```
> (let ((a (+ 4 7))                           ;; wrong!
        (b (* a 5)))
    (+ a b))
```

Don't think that a gets the value 11 and therefore b gets the value 55. That let expression is equivalent to defining a helper procedure

```
(define (helper a b)
  (+ a b))
```

and then invoking it:

```
(helper (+ 4 7) (* a 5))
```

The argument expressions, as always, are evaluated *before* the function is invoked. The expression (* a 5) will be evaluated using the *global* value of a, if there is one. If not, an error will result. If you want to use a in computing b, you must say

```
> (let ((a (+ 4 7)))
    (let ((b (* a 5)))
      (+ a b)))
66
```

⇒ Let's notation is tricky because, like cond, it uses parentheses that don't mean procedure invocation. Don't teach yourself magic formulas like "two open parentheses before the let variable and three close parentheses at the end of its value." Instead, think about the overall structure:

```
(let variables body)
```

Let takes exactly two arguments. The first argument to let is one or more name-value groupings, all in parentheses:

```
((name1 value1) (name2 value2) (name3 value3) ...)
```

Each name is a single word; each value can be any expression, usually a procedure invocation. If it's a procedure invocation, then parentheses are used with their usual meaning.

The second argument to let is the expression to be evaluated using those variables.

Now put all the pieces together:

```
(let ((name1 (fn1 arg1))
      (name2 (fn2 arg2))
      (name3 (fn3 arg3)))
  body)
```

Boring Exercises

7.1 The following procedure does some redundant computation.

```
(define (gertrude wd)
  (se (if (vowel? (first wd)) 'an 'a)
      wd
      'is
      (if (vowel? (first wd)) 'an 'a)
      wd
      'is
      (if (vowel? (first wd)) 'an 'a)
      wd))

> (gertrude 'rose)
(A ROSE IS A ROSE IS A ROSE)

> (gertrude 'iguana)
(AN IGUANA IS AN IGUANA IS AN IGUANA)
```

Use `let` to avoid the redundant work.

7.2 Put in the missing parentheses:

```
> (let pi 3.14159
       pie 'lemon meringue
    se 'pi is pi 'but pie is pie)
(PI IS 3.14159 BUT PIE IS LEMON MERINGUE)
```

Real Exercises

7.3 The following program doesn't work. Why not? Fix it.

```
(define (superlative adjective word)
  (se (word adjective 'est) word))
```

It's supposed to work like this:

```
> (superlative 'dumb 'exercise)
(DUMBEST EXERCISE)
```

7.4 What does this procedure do? Explain how it manages to work.

```
(define (sum-square a b)
  (let ((+ *)
        (* +))
    (* (+ a a) (+ b b))))
```